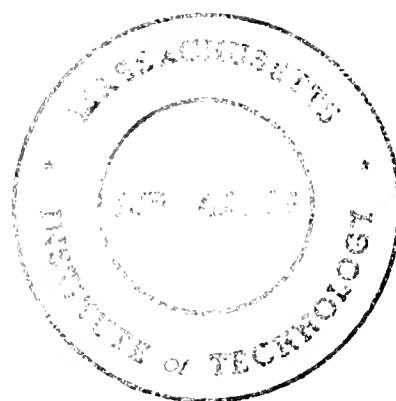GROWTH OPPORTUNITIES AND

CORPORATE INVESTMENT THEORY

IN EFFICIENT FINANCIAL MARKETS

by

James L. Paddock

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GROWTH OPPORTUNITIES AND CORPORATE INVESTMENT THEORY IN EFFICIENT FINANCIAL MARKETS*

Abstract

In this research we analyze the components of the firm's investment opportunity set and the linkages between financial market valuation of those growth opportunities and their intertemporal realization through capital investment in the real markets. We decompose the investment process into a two equation simultaneous system and find that shifts of the investment opportunity schedule, as reflected in changes in the financial market valuation of the firm, can be used as a predictor of the firm's investment program.

Investment opportunities available to a firm are reflected in the financial market valuation of that firm. The purpose of this research is to analyze the relationship between that market value and the investment decisions of those corporations. In particular, we find that changes in a firm's market value can be used as a predictor of its investment behavior. This is because a value-maximizing firm must select an optimal, intertemporal investment program which simultaneously depends on the real productivity characteristics of assets and the financial market valuation of claims to the cash flow streams generated by those assets. We show that an understanding of the linkages between the financial markets and the "real" markets is crucial to an analysis of firm investment decisions. Shifts in the firm's investment opportunity schedule are captured immediately by changes in the firm's efficient market valuation; and the firm adjusts its real investment program as a distributed lead into the future in response to these shifts. We decompose the investment process so as the examine explicitly the simultaneity between the firm's investment decisions and the opportunity set.

The theoretical development and empirical analysis of investment behavior by firms was the basis of considerable economic research during the past two decades, culminating in two major branches: the "flexible-accelerator" theory

and the "Tobin's q" theory. The flexible-accelerator model,[1] originally

introduced by Chenery [5] and Koyck [22], attempted to incorporate the time

patterns of investment behavior in the neoclassical theory. However, this model

suffers from several important weaknesses, besides not explicitly incorporating

financial market information, including: (a) the distributed lag of investment

follows from an ad hoc specification of the accelerator; (b) output level is set

exogenously rather than as a choice variable of the firm; (c) "desired" capital

stock is ill-defined and specified in a static context.[2]

Another line of inquiry into investment behavior hypothesized that if the

value of an asset in the financial market is higher than its cost in the capital

goods' market, real investment in that asset will occur. That is the essence of

what has come to be called the Tobin's q theory of investment. While this

approach dates to Keynes,[3] Grunfeld [13], in a seminal piece, presented the

first formal treatment in the literature of the q concept. He analyzed the

reasonably successful role of profits in earlier studies as a determinant of

investment. He found that profits was a surrogate for other, more basic

variables and concluded,

> "The principal variable of this type is the 'market value of the
> firm,' that is, the value placed upon the firm by the securities
> markets. When taken in conjunction with an estimate of the
> replacement value of the physical assets of the firm, this variable
> appears to be a sensitive indicator of the expectations upon which
> investment decisions are based. Furthermore, and in part because of
> lags in the process of physical investment, this variable signals
> changes in investment before they actually occur."[4]

Tobin [35] and Brainard and Tobin [4] developed a general equilibrium macro

model which included a specific financial and monetary sector, and presented the

theoretical derivation of the q concept, relating it explicitly to government

policy decision variables.[5] One of the basic hypotheses in their model is that

the ratio q, defined as the market valuation of equities relative to the

replacement cost of the physical assets they represent, is the major determinant of new investment.

In an important insight, Lintner [24] explicitly recognized that a distributed lag theory of investment must be based on those factors which "represent essential determinants of the position--and of the shifts in the position"[6] of the Fisherian investment opportunity function. In Fisher's certainty world, these basic factors include both "real" market variables and financial market variables. We pursue this investigation in our analysis which follows. We present a theoretical analysis of the investment opportunity set from which time lags are derived, rather than specified ad hoc as in most earlier research. Our analysis yields a two equation, simultaneous system as our investment model. These two equations reflect a decomposition of the investment opportunity set. This simultaneity also has important implications for the empirical analysis of investment since previous investment models have been only single-equation, and often use a static methodology.

I. The Determinants of Firm Value

The value of the firm is comprised of both physical and intangible assets. We define equilibrium firm market value, V, as: $V \equiv V_p + V_i$, where V_p and V_i are the equilibrium values of the firm's physical and intangible assets, respectively. The equilibrium value of any particular asset is defined as the time integral of the discounted cash flows it is expected to generate, either by itself or by making an incremental contribution to cash flow when used in combination with other assets. If perfect secondary markets exist for either physical assets or intangibles, equilibrium asset value is the observed price of that asset. Both V_p and V_i, and therefore V, are stochastic over time. For a specific firm they change due either to exogenous reasons or to the purchase or

sale of assets by the firm.

Physical assets are plant, equipment or structures and are often referred to as "real capital stock." Physical assets are already "in-place," generating current cash flows; they are expected to generate cash flows in the future. If either the discount rate or expected future cash flows to these currently existing assets changes, e.g., due to a change in consumer preferences for the output of this physical asset, then there will be a change in the asset's secondary market value resulting in a $\Delta V_p \neq 0$.

Intangible assets are generally firm-specific, and although they are suppressed in most of the literature, we consider them explicitly. Intangibles also derive their value from an inherent capability to generate cash flows, whether alone or in combination with physical assets. Obviously this category of intangibles includes many things. In particular, the class of intangibles includes any aspect which implies an excess return (i.e., rents) either to a physical asset or another intangible asset. The market's identification of any form of excess return is instantaneously reflected in V_i. For example, monopoly power derived from earlier research and development expenditures is reflected as expected rents in the cash flows to that firm's existing physical assets. This market power is an intangible asset, and its exploitation represents a project with NPV > 0. The existence of expected rents is a necessary condition for a project to have a positive NPV; the NPV of these rents enters ΔV through $\Delta V_i > 0$.

In general, physical and intangible assets augment each other in the generation of cash flows. Thus the projects a firm undertakes will probably involve investment in a combination of intangible and physical assets. Rarely do we observe physical assets without concomitant intangible ones. However, some intangibles need not be combined with a physical one. For example, the value of

a consulting firm will be primarily intangible assets, with only incidental
physical ones. Other examples of intangible assets include patents, distribution
networks, monopoly power, management expertise, and growth opportunities--the
latter is the focus of our investigation.

An interesting subclass of intangible assets is profitable new projects or
"growth opportunities." These growth opportunities require investment at some
point in either physical or intangible assets (or both). Investment is a
necessary condition for their cash flows to be realized. This distinction leads
to the following decomposition of intangible assets: $V_i \equiv V_{ie} + V_{ig}$,
where V_{ie} and V_{ig} are the equilibrium market values of existing intangible
assets and all growth opportunities, respectively. Therefore $V \equiv V_p + V_{ie} + V_{ig}$.
Existing intangible assets are defined as those which the firm already owns or
can buy; but in either case are currently producing cash flows. Growth
opportunities reflect future cash flows which will be generated only if the
requisite investment program is carried out.

Growth opportunities occur in two general forms:

(1) those projects which require an immediate investment decision based on
     the NPV criterion; and,

(2) those which are an option to invest in future projects having cash flows
     (including the investment) which will start (be "exercised") at some
     future time.

Either form (1) or (2) above may or may not be directly related to the firm's
current asset base. Both forms may instantaneously occur to a specific firm due
to exogenous factors. Obviously, V_{ig} is a portfolio of options on different
projects. These options may mature at different points in time, and some may be
exercised independently of others. The value of this portfolio reflects the lag
structure of these opportunities and their maturation process. We term this

portfolio the "investment opportunity set" (IOS).

In an efficient market, any change in the value of these growth opportunities is reflected immediately in the market value of the firm's securities. When investment projects, immediate or future, are identified with a positive NPV, there will be an immediate value increment to the firm through $\Delta V_{ig} > 0$. If the estimate of NPV changes, we can observe $\Delta V_{ig} \lessgtr 0$. But we always will have $V_{ig} \geqq 0$ as with any option. Identified negative NPV projects will not affect the firm in the form of a value decrement because the firm is not forced to take these projects, and the value-maximizing firm will choose not to. This decision is anticipated by the market just as it anticipates the firm will always accept projects with NPV > 0. A "negative NPV project" in this case does not include unanticipated downward revisions of expected future cash flows from existing assets, and upward revisions of their discount rate. Value decrements from those revisions enter V through ΔV_p and ΔV_{ie}.

Clearly, asset valuation categories are not unrelated. For example, an unexpected enhancement of monopoly power may be reflected by ΔV_p, ΔV_{ie}, and also ΔV_{ig} if it calls for additional investment, e.g., in production capacity.

Our interest is in investment and thus V_{ig}, which represents the investment opportunity set facing the firm. From the foregoing definitions, the following is clear: investment by the firm will occur only when $V_{ig} > 0$. Figure 1 below illustrates these concepts for a simple static case. In Figure 1, the horizontal axis measures the firm's existing capital asset base in units of capital, K, at a given point in time. This capital base includes both physical and intangible assets and their secondary market value is: $V_K = V_p + V_{ie}$. Assume these assets to be homogeneous, and let P be their per unit price in the secondary market, thus $V_K = PK$.

Line PP in Figure 1 represents this unit price. Line VV is the current IOS

measured in dollars per unit capital. The spread between VV and PP is exactly the NPV of currently maturing projects. Thus the firm moves down VV by taking on projects, i.e., investing in K, until point point K^*_0 is reached. K^*_0 is the optimal capital stock and represents that point at which NPV = 0 at the margin, i.e., value added equals cost. All projects to the left of K^*_0 have NPV > 0. The total value of the firm is the left integral, from K^*_0, of VV--with that portion above PP representing rents.

We can illustrate $\Delta V_{ig} > 0$ by a shift in the investment opportunity set to V'V'. This change requires new investment of amount $K^*_1 - K^*_0$ as now there are additional current investment projects with NPV > 0. Thus $\Delta V_{ig} > 0$ implies $\Delta K > 0$. K^*_1 is the new optimal capital base.
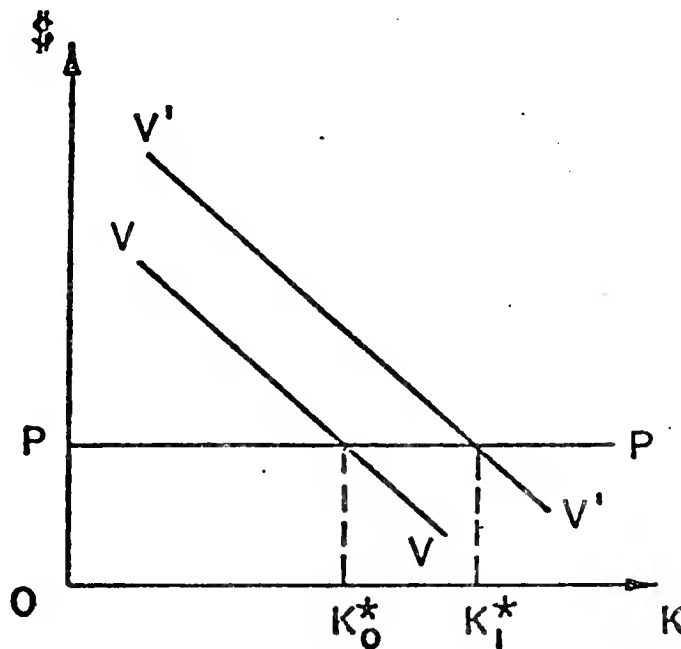

Figure 1 The Firm's Investment Opportunity Schedule as of Time t

II. The Analytics of Firm Investment

Our purpose is to analyze the optimal dynamic process by which the firm exercises its growth opportunities through investment expenditures. Growth

opportunities for a firm may decay over time if not realized via investment.
$V_{ig} > 0$ is a necessary condition for investment to occur. Thus the ratio

$$\frac{V}{V_K} = \frac{V_p + V_{ie} + V_{ig}}{V_p + V_{ie}}$$

will exceed unity as a precursor to a firm's program of investment. Recall that
the denominator, V_K, includes only the firm's current income-producing assets.
Thus this ratio is a measure of the firm's growth opportunities as valued by the
financial markets.

In addition, we assume that there are costs incurred whenever the firm
adjusts its capital base. These adjustment costs are of two forms:

(1) Those which are internal to the firm. For example, it may be necessary
     to shut down a product output operation for a time while installing new
     machines; and

(2) Those which are due to imperfections in the secondary market. For
     example, in a thin (illiquid) market, the delivery time for a physical
     asset may be decreased only at a higher cost.

If these costs of adjustment[7] are an increasing function of the relative rate
of change in the capital base, then a firm's optimal investment program will be
seen to be a distributed lead over future time periods. In terms of Figure 1,
K^*_1 will not be reached in the current time period. Without these costs,
capital stock adjustment occurs instantaneously and the firm behaves in a myopic
fashion. The NPV calculation, and therefore V_{ig}, must obviously take account
of any adjustment costs associated with the project.

A. The Dynamics of Firm Investment

We now develop a model which captures the dynamic investment process
discussed in the previous section. Our firm value function will include both the
effects of cumulative existing assets over time and the effect of intertemporal

decisions on V_{ig}. The latter decisions dictate the time path of investment.

Define:

K_t ≡ the number of units of the firm's homogeneous capital stock (tangible plus existing intangible assets) at the beginning of period t.

$\dot{K}_t$ ≡ $\frac{\partial K}{\partial t}$ ≡ the flow (in units) of net investment[8] during period t.

k_t ≡ $(\frac{\dot{K}}{K})_t$ ≡ the rate of investment in period t relative to the existing stock of capital.

P_t ≡ the market price of a unit of capital at the beginning of period t.

ρ_t ≡ the market-required rate of return on the firm during period t.

π_t ≡ the set of parameters which determines net, after-tax operating revenue flow (before debt service) to capital during period t.

R_t ≡ $R_t(K_t, \pi_t)$ = the dollar amount of the net, after-tax operating revenue stream to capital in period t, characterized by $R_K > 0$, $R_{K\pi} > 0$, $R_\pi > 0$, where subscripts denote partial derivatives.[9]

$G(k)$ ≡ the dollar cost of adjustment function per unit capital dependent on the relative rate of investment. Therefore, total costs of adjustment equal $\dot{K}G(k)$.

Let the per unit adjustment cost be an increasing (strictly convex) function, $G(k)$, of the absolute value of the relative rate of investment where $G(0) = 0$, and: sign $G'(k)$ = sign k for k ≠ 0, $G''(k) > 0$, where primes denote partial derivatives. We also assume this adjustment cost function to be additively

separable from the revenue function. In growth models, adjustment cost functions

specified in the absolute level of investment, $\dot{K}$, may clearly become

inconsequential over time as K grows; whereas specifying them as a function of

the relative rate of investment, $\dot{K}/K$, has meaningful economic content as

$t \longrightarrow \infty$.

The market-determined value of the firm at time zero, V_0, including present

and future investment opportunities is equation (1).[10]

$$V_0 = \int_0^\infty [R - \dot{K} G(k) - P \dot{K}]e^{-\rho t}dt \qquad (1)$$

Management's objective is to maximize firm value to the current shareholders, that

is, Max V_0 in equation (1) subject to[11] $\dot{K} \geqq 0$, where $\dot{K}$ is the control variable.

The Hamiltonian can be formulated as:

$$H = [R - \dot{K}G - P\dot{K} + v\dot{K}]e^{-\rho t} \qquad (2)$$

The necessary conditions for a maximum are:

$$\frac{\partial H}{\partial \dot{K}} = 0 = [- G - kG' - P + v]e^{-\rho t} \qquad (3)$$

$$-\frac{\partial H}{\partial K} = \dot{v} = \rho v - R_K - k^2 G' \qquad (4)$$

where v is the shadow price for capital.

Following Tobin in [35] and [4], average "q" is defined as the ratio of the

total market value of the firm and the replacement cost[12] of all the firm's

physical assets, i.e., $q \equiv \frac{V}{V_p}$. However, our interest is in "marginal q," a

concept we define as the ratio of: (a) the change in firm market value at the

margin due to a one-unit increase[13] in capital input, and (b) the cost of

acquiring that additional unit. Representing marginal q by q', we can define it

in our current model as:

$$q' = \frac{v}{P} \tag{5}$$

Thus $q' \gtreqless 1$ as $P \lesseqgtr v$. Note that the net present value (NPV) is therefore indicated by q' as follows: NPV $\gtreqless 0$ as $q' \gtreqless 1$.

"Average q," denoted by $q = (R/PK)/\rho$ is the average rate of return, (R/PK), earned by the firm on its total capital base relative to the market cost of capital, ρ. This definition of q is identical to total market value of the firm, V, relative to the reproduction cost of the total capital base, PK ($\equiv V_\rho$). Thus:

$$q = \frac{(R/PK)}{\rho} \neq \frac{v}{P} = q'$$

The salient point is that average and marginal q may differ in all cases except long-run, steady-growth equilibrium with homogeneous capital (i.e., no technological change) and no rents.

Rearranging terms in (3) and substituting q' from (5) let us write:

$$q' = 1 + \frac{G}{P} + \frac{kG'}{P} \tag{6}$$

Equation (6) states that at the point of value maximization the firm will choose investment such that marginal value equals marginal cost, where the latter includes adjustment costs. We analyze the interesting composition of this marginal cost term below. From (6) we see that $q' = 1$ when there are no costs of adjustment as expected.

Value-maximizing firms will always, and instantaneously, invest down[14] their investment opportunity schedule in all projects with $q' \geq 1$, that is, (NPV) ≥ 0, to the point where investment cost just equals its marginal value, i.e., $(\partial V)/(\partial \dot{K}) = 1$ and $q' = 1$. For all projects without adjustment costs, this amount of investment would occur instantaneously and $q' = 1$ would hold

continuously to avoid arbitrage. However, the existence of costs of adjustment

precludes the amount of instantaneous investment required[15] to set q' = 1, at

least in the short run.

As adjustment costs (which are deductions from net marginal revenue) cause q' to

remain greater than unity, $\dot{K}_t$ will be less than the amount which would prevail

if there were no adjustment costs.

We also can solve for $\dot{q}'$ which is the time path of q' for a <u>fixed</u> IOS.

Dividing (4) through by P and substituting[16] for $\dot{q}' = \frac{\dot{v}}{P}$ yields the equation of

motion:

$$\dot{q}' = \rho q' - \frac{R_K}{P} - \frac{k^2 G'}{P} \tag{7}$$

We analyze the implications of (7) for the firm's investment behavior below.

B. <u>Investment in Growth Opportunities in a Dynamic System</u>

At any point in time, t, the firm chooses a rate of investment, k_t, so as

to satisfy (6); k_t is the control variable. These changes in the capital stock

move the firm <u>along</u> the fixed investment opportunity schedule, thus inducing

feedback effects on q'_t, as captured by (7), with concomitant implications for

the next period's investment choice. Since G(0) = 0, equation (6) reveals that

q' will remain equal unity when there is no ongoing firm investment. When there

exist growth opportunities with NPV > 0 however, q' reflects that fact and will

exceed unity by an amount which can be decomposed as follows, and a positive rate

of investment will result.

The total cost of a given flow of investment at time t is:

Total cost of investment = $P\dot{K} + \dot{K}G(k)$ $\tag{8}$

Thus the marginal cost of an extra unit of investment is the partial derivative of

equation (8) with respect to $\dot{K}$ or:

$$\frac{\partial(\text{Total Cost})}{\partial \dot{K}} = 1 + \frac{G}{P} + \frac{kG'}{P} > 1 \qquad (9)$$

where we have normalized on P. Equation (9) is the marginal cost of investment and accounts for the three terms on the right-hand side of (6), which represents that portion, in equilibrium, of the investment costs which must be "charged" against period t value additions. We see from (9) that the relevant marginal cost is greater than P(=1) due to the adjustment cost terms. Thus a value–maximizing firm will indeed reach a point where it will not increase the rate of investment even though the marginal value product exceeds P alone.

Further, K changes intertemporally also if there is nonzero net investment. Thus for a given net investment rate, $\dot{K}$, the increase in K implies a decrease in k and thus a decrease in the next period's per unit adjustment cost, G(k). This follows from the partial derivative of equation (8) with respect to K:

$$\frac{\partial(\text{Total Cost})}{\partial K} = -G'k^2 < 0 \qquad (10)$$

Equation (10) represents the total "savings" in adjustment costs due to an increase today in the capital stock for a given future investment rate. This accounts for the third term on the right-hand side of (7). These "savings" raise the firm's optimal $\dot{K}$ from what would otherwise be the case. Accounting for these future savings represents a component of nonmyopic behavior since the firm will modify its current period investment decision in light of its effects on the future IOS.

Investment Along a Fixed Opportunity Locus

Derivations in the literature can be shown to have relationships in which $\dot{K}_t$ is negatively related to q', i.e., $\frac{\partial \dot{K}}{\partial q'} < 0$, e.g., Ciccolo [6], and others in which $\frac{\partial \dot{K}}{\partial q'} > 0$, e.g., Abel [1] and Tobin [35]. This seeming anomaly stems from two

sources: (a) the need to draw a distinction between average and marginal q; and (b) the need to distinguish between shifts <u>along</u> a fixed investment opportunity schedule and shifts <u>of</u> the schedule itself. We resolve this anomaly in the following analysis. The value-maximizing firm chooses optimal $k^{\star}_t$ at each point in time so as to satisfy (6). But each additional unit of capital decreases q' at the margin as the firm moves down the stationary IOS accepting projects with lower NPV. Taking $(\partial q')/(\partial K)$ in (6) (and evaluating it for $\dot{K} > 0$) we see this negative effect on q' in (11):

$$\frac{\partial q'}{\partial K} = - \frac{k(2G' + kG'')}{PK} < 0 \qquad (11)$$

This effect can be illustrated by a numerical example applied to (7), the equation for motion of q' along a fixed IOS where we expect $\dot{q}' < 0$ when $\dot{K} > 0$. We also also know that $(R_K/P) > \rho$ since it must cover the adjustment costs as well as the market-required cost of capital. Assume $\rho = .1$, q' = 1.03, R_K = 4.25, P = 40$[\Longrightarrow (R_K/P) > \rho]$, $G = k^2$, and k = .1. Then from (7) we have q' = −.0033 < 0 as expected. Figure 2 illustrates this process. Where VV, the IOS, intersects the cost of investment schedule, PP, at point A we have q' = 1. Thus the firm chooses K_0 as its optimal capital stock. As it moves out from 0 to K_0 by investing, it is moving down the VV schedule, accepting projects with NPV > 0 (and therefore q' > 1) until point A where NPV = 0 (and q' = 1). Thus we see how these shifts <u>along</u> (up and down) the fixed schedule VV imply $\partial q'/\partial K < 0$. Among other things, the fixed VV schedule implies a constant ρ, P, and π, assumptions we will relax later. A horizontal PP curve implies competitive secondary markets for assets and no costs of adjustment.

Now consider incorporation of the simple quadratic costs of adjustment function in our numerical example above which yields cost curve PP' in Figure 2 and capital stock of $K_1 < K_0$ at point B where q' = 1 + G/P + (kG')/P from
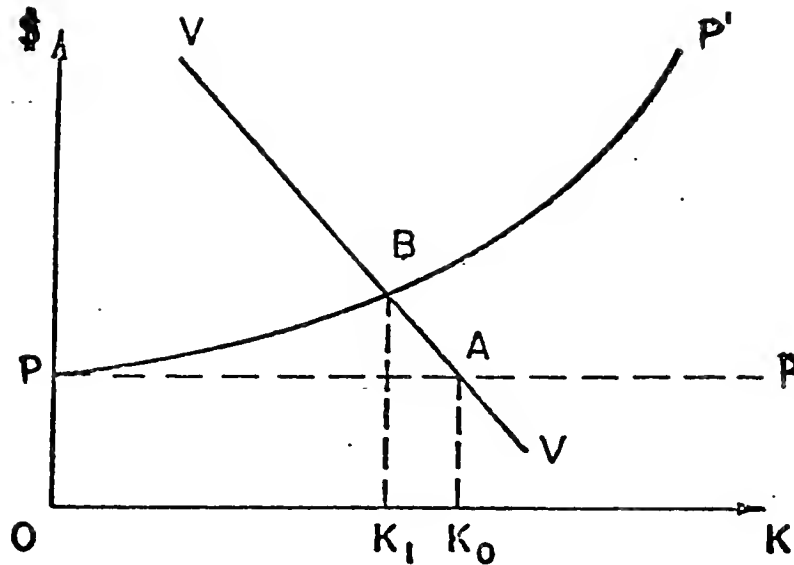
equation (9).

Figure 2 Fixed IOS as of Period t With (PP) and Without (PP') Strictly Convex
Costs of Adjustment

(Note: This figure represents the model as of a given period, t, e.g., PP'
illustrates the costs of adjustment given that the firm adjusts K in period
t.)

In Figure 2, the NPV of the firm at point B is equal to the integral of the area

under VB less the integral under PB. This value is strictly less, by the area

PBA, than the firm without adjustment costs. This analysis has assumed no

exogenous changes in q'. We have been analyzing only movements along the

investment opportunity schedule as the firm moves toward K*. We have not

considered any intertemporal shifting of the IOS.

Thus, past values of q' have a deterministic effect on investment in present

and future periods. The practical implications are obvious. In the certainty

model of Fisher [11], the firm need only observe the market rate of interest in

making its investment decisions. In our model with costs of adjustment, the firm

need only observe q' in making its investment decisions. The decision rule is:

if $q' > 1 + g'$, invest, where $g' \equiv (G/P) + (kG'/P)$.

Investment Response to a Shift of the Opportunity Locus

We will now consider the firm's investment decision when the investment opportunity schedule itself <u>shifts</u>. The analysis in the previous section only considered one part of a simultaneous dynamic system. The idea of a <u>fixed</u> intertemporal opportunity locus is only a partial equilibrium approach and assumes a constant π, P, and ρ. But an exogenous change in any or all of these parameters will cause the investment opportunity schedule, and therefore q', facing the firm in that particular period to shift up or down triggering changes in the investment program. For example, suppose π decreases (uniformly at all levels of K). Then for every level of K, net revenues will be smaller and, assuming constancy of the costs of investment locus, firm value will be less. Likewise, an increase in π will imply a larger firm value, <u>ceteris paribus</u>.

To derive the effects of these shifts in q' we differentiate (3) with respect to time to yield:

$$\dot{v} = \dot{k}(2G' + kG'') \tag{12}$$

The right-hand side of (12) represents the change in marginal cost over time. This can be shown by taking the partial derivative of marginal cost (equation (9)) with respect to time:

$$\frac{\partial(\text{Marginal Cost})}{\partial t} = \dot{k}(2G' + kG'') \tag{13}$$

Thus (12) represents the dynamic equilibrium condition that investment choice is made such that the change in marginal value is equated to the change in marginal cost.

Substituting (12) into (4), dividing through by ρP, and substituting for q' yields:

$$q' = \frac{R_K + k^2 G' + \dot{k}[2G' + kG'']}{\rho P} \qquad (14)$$

Our concern is with the comparative dynamics analysis with respect to changes in π, ρ, and P because it is obvious now that, <u>simultaneously</u> with $\dot{K}$, changes in q' can occur due to changes in π, ρ, and P as follows from (14):

$$\frac{\partial q'}{\partial \pi} = \frac{R_{K\pi}}{\rho P} > 0 \qquad (15)$$

$$\frac{\partial q'}{\partial \rho} = -\frac{R_K + k^2 G' + \dot{k}[2G' + kG'']}{\rho^2 P} < 0 \text{ for } \dot{k} \gtreqless 0 \qquad (16)$$

$$\frac{\partial q'}{\partial P} = -\frac{R_K + k^2 G' + \dot{k}[2G' + kG'']}{\rho P^2} < 0 \text{ for } \dot{k} \gtreqless 0 \qquad (17)$$

Equations (15)-(17) are an important result. They tell us that an increase in π, for example, <u>ceteris paribus</u> and without costs of adjustment, should trigger an (instantaneous) increase in the firm's capital stock such that q' remains equal to unity. This shift in the IOS is reflected in the market value of the firm and is clearly illustrated[17] in Figure 3 as follows. The exogenous shock, dπ > 0, shifts VV up to V'V'. Thus for fixed capital stock, K_0, point A moves up to A'. But now q' > 1 at A', and the firm will now invest so as to increase the capital stock from K_0 to K'_0 at point D where again we have q' = 1. This investment now has increased the value of the firm by the integral of the triangle AA'D. It is in this important sense that q' > 1 stimulates and leads investment. Without costs of adjustment the investment amount, $K'_0 - K_0$, would occur instantaneously. With positive costs of adjustment we have a similar result in Figure 3 on cost curve PP'. For the same level of capital stock, K_1, firm value has increased by the amount of the integral of the

upper-left parallelogram formed by VBB'V'. The increase in π now has caused an increase in q' from point B such that at point B' we have q' > 1 + g'. This causes the firm to invest amount $K'_1 - K_1$ until point E is reached where again q' = 1 + g' and firm value has increased by triangle BB'E. At capital stock level K_1, the difference in value, BB'E represents an intangible asset or growth opportunity which can only be realized through real investment so as to increase the capital stock to level K'_1. The market value of the firm already reflects these "rents" as indicated by q' > 1. Now we can see clearly the distinction between changes in q' due to shifts along VV (to points on either side of B) and shifts of VV (to points on V'V' such as B' and E). This important distinction has not been shown in the literature. Note by inspection of Figure 3 that with positive costs of adjustment, the optimal capital stock and investment rate, and maximum firm value will be strictly less than is the case with no adjustment costs.
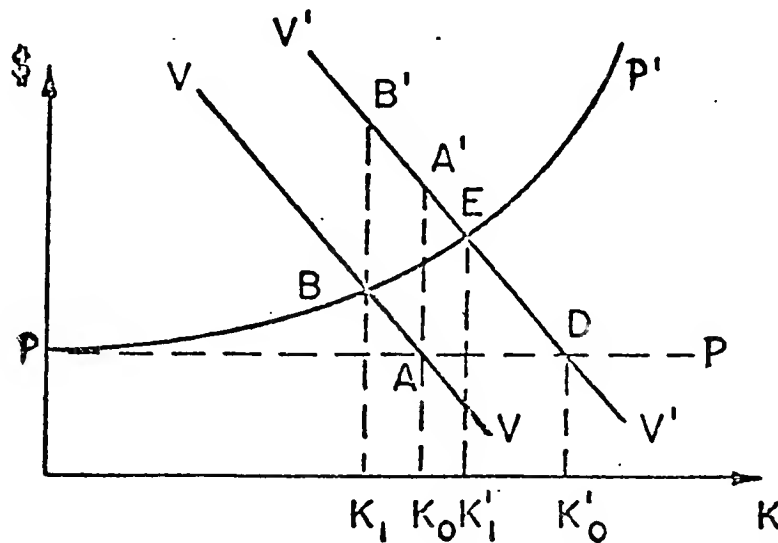

Figure 3 Shift in the IOS as of Period t with Strictly
Convex Costs of Adjustment

It remains to be shown how the IOS shifts derived in equations (15)-(17) impact on the rate of investment. We will show that changes in q' cause changes

in the rate of firm investment, which in turn affect q' in a simultaneous system. Further, we can tract these investment rate changes over time as a distributed lag of q'.

To understand the process we first present a numerical example and then show the formal derivation. Consider equation (3) which can be rewritten as:

$$(q' - 1)P = G + kG' \tag{18}$$

Assume the quadratic[18] form for G such that $G = Ak^2$ where A is simply a parameter $(A > 0)$ that translates the k relationship in the costs of adjustment into unit dollar costs. Substituting for G in (18) and solving for k gives:

$$k^* = [\frac{(q' - 1)P}{3A}^{1/2}] \tag{19}$$

where k^* denotes the optimal value.

To see the effect on k of a change in q' we have from (19):

$$\frac{\partial k^*}{\partial q'} = \frac{1}{2} (\frac{P}{3A})^{1/2} (q' - 1)^{-1/2} \quad \begin{cases} > 0 \text{ for } q' > 1 \\ = 0 \text{ for } q' = 1 \end{cases} \tag{20}$$

Thus shifts in the IOS affect k through their effect on q'; and k varies directly with q' as it varies around its equilibrium value of unity. From (15)-(17) this implies that an increase in project profitability will increase the rate of investment, while an increase in the market-required cost of capital or the cost of investment goods will decrease the rate of investment, ceteris paribus. This relationship can readily be seen in a numerical example applied to (3) and (19), shown in Table I below. There for differing values of q' (in either equation) we solve for the corresponding value of k.

q' =	1	1.0075	1.03	1.27	4
k* =	0	.05	.1	.3	1

Table I   Values of q' Necessary to Generate Various Levels of Investment With Quadratic Adjustment Costs (assumes $\rho = .1$)

Note that higher values of q' draw forth higher values of k as predicted.

We can now show this same result formally in the equation of motion for k derived in our model. Solving (14) for $\dot{k}$ gives us the equation of motion for the rate of investment as:

$$\dot{k} = \frac{\rho P q' - R_K - k^2 G'}{2G' + kG''} \tag{21}$$

The comparative dynamics [19] reflecting the effect on k of a shift in the IOS yield from (21):

$$\frac{\partial \dot{k}}{\partial q'} = \frac{\rho P}{2G' + kG''} > 0 \tag{22}$$

as expected.

Thus the two equation simultaneous system we have derived as our investment model is:

$$\dot{q} = \rho q' - \frac{R_K}{P} - \frac{k^2 G'}{P} \tag{7}$$

$$\dot{k} = \frac{\rho P q' - R_K - k^2 G'}{2G' + kG''} \tag{21}$$

This model reflects the determination of a firm's investment rate as a function of both movement along an IOS and shifts of the IOS. Only one instance of q' ≠ 1 (i.e., a shift in the IOS), with strictly convex costs of adjustment, is necessary to trigger an infinite play-out of investment. Further instances of departure of the level of $q'_{\gamma > t}$ from its equilibrium decay path serve simply to dictate the new level of $k^*_{\gamma > t}$ for its new path. These paths indicate the intertemporal regression of q' towards unity, k towards its steady state value, and $\dot{k}^*$ towards zero.

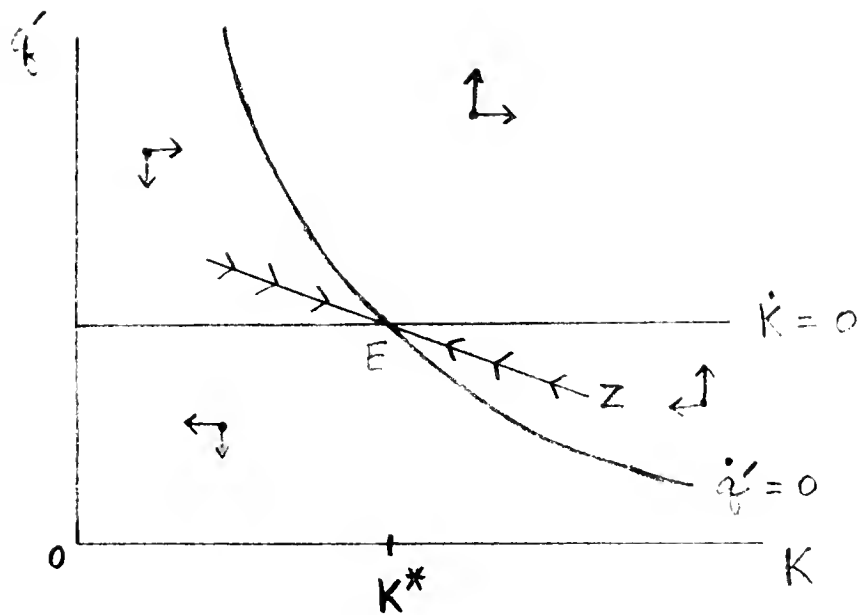The phase diagram[20] of this system in Figures 4 and 5 illustrates these

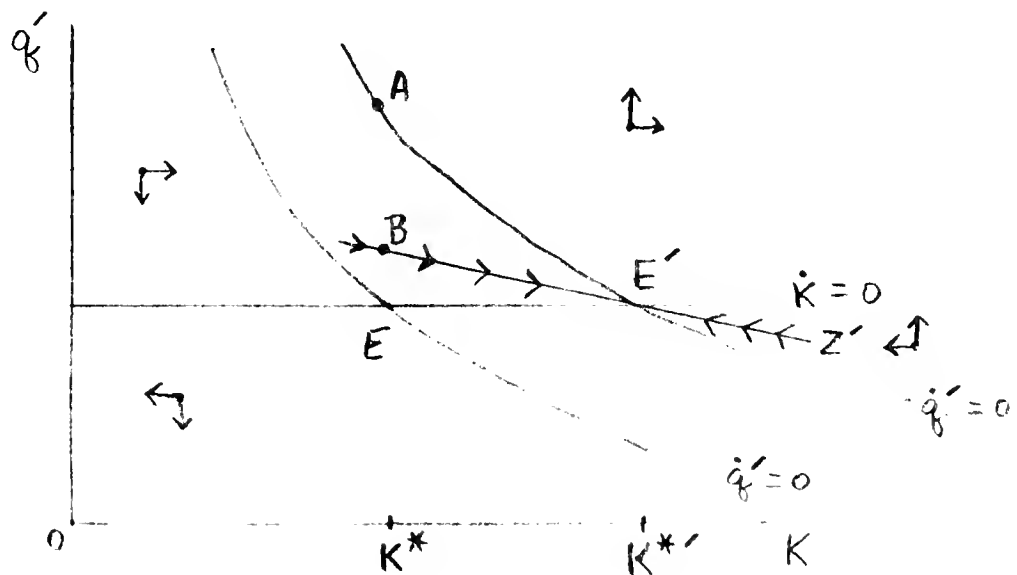Figure 4. Phase Diagram for a Fixed IOS with Convex Adjustment Costs

Figure 5. Phase Diagram for a Shift in the IOS with Convex
Adjustment Costs.

comparative dynamics and the stability properties of our simultaneous system.

The arrowed line, Z, in Figure 4 indicates the saddle point path for convergence

to K* for a fixed IOS, the latter represented by the $\dot{q}' = 0$ locus. Under perfect

competition with all projects expected to earn a competitive return, and thus

their NPV = 0, path Z would coincide with the $\dot{q}' = 0$ locus. In Figure 4 we

assume there are some IOS projects characterized by NPV > 0, i.e., expected to

earn rents; thus path Z lies off $\dot{q}' = 0$ as these projects are taken on until

NPV = 0 projects are reached at point E for equilibrium K*.

Now consider a rightward shift in that IOS due to, for example, $d\pi > 0$. This

case is illustrated in Figure 5, where without adjustment costs the capital stock

would adjust instantaneously to K*' at E'. (With no investment the system would

move to point A.) However, with convex adjustment costs the firm moves to point

B and spreads investment out into the future as the projects it takes on move it

along Z' from B to E'. The location of Z', and thus the extent of the jump from

E to B (towards A) will depend on the specification of the costs of adjustment

function. Figure 5 illustrates the convergency of the system to the stable

equilibrium at E' where q' = 1 and K*' represents a determinate firm size at an

instant of time for a stationary IOS and G(k) cost function.

We can now present a testable form of the hypothesis of our model to which

simultaneous equation econometric techniques can be applied.

We can formally derive k as a distributed lag of q' as follows. Substituting

from (7) into (21) we obtain:

$$\dot{k} = \frac{P\dot{q}'}{2G' + kG''} \tag{23}$$

We can write (23) in discrete form as:

$$k_t^* = \frac{Pq_t' - Pq_{t-1}'}{2G'(k_t) + k_t G''(k_t)} + k_{t-1}^* \tag{24}$$

But likewise we know that:

$$k^*_{t-1} = \frac{Pq'_{t-1} - Pq'_{t-2}}{2G'(k_{t-1}) + k_{t-1}G''(k_{t-1})} + k^*_{t-2} \qquad (25)$$

Substituting backwards for all t-1 from (25) into (24), and assuming the system was in equilibrium at time zero, yields:

$$k^*_t = F_t[q'_t, q'_{t-1}, \ldots, q'_1] \qquad (26)$$

where F_t is some polynomial distributed lag in q'. Equation (26) is a key result. It is a theoretical justification for real investment outlays by firms to be determined by a distributed lag of real and financial market variables. (Equivalently, the level of q' today causes an infinite distributed lead relationship with future investment.) This important result is derived from the structure of our model, rather than specified ad hoc as by earlier researchers, such as in flexible-accelerator models.

III. CONCLUSION

Efficient financial markets recognize investment opportunities facing firms in the real goods' markets, and those opportunities are reflected immediately through changes in the financial valuation of those firms. The analysis of these linkages brings out several theoretical points which have not been shown before in the literature. First, we separate the investment opportunity set from the total asset structure of the firm. We show how these opportunities in the real markets are valued in efficient financial markets. By incorporating this valuation process, the real investment behavior of the firm can be predicted as a distributed lag on marginal q as reflected in stock market data.

Second, we decompose the firm's investment opportunity set into those elements affected by exogenous shifts in the factors which determine that set and

those affected by "take-downs" of specific opportunities from the set. We derive the firm's actual investment behavior over time based on the changing composition of this opportunity set.

Third, the various effects of adjustment costs on firm investment behavior are analyzed. The structure of these costs gives rise to a determinate firm size and a distributed lag pattern of real investment.

The empirical implications of these theoretical results are important. The distributed lag relationship of investment and changes in firm market value is specified explicitly by the model. This avoids the severe criticism of earlier ad hoc specifications of the flexible-accelerator. We have indicated how financial market data may help determine a direct empirical measure of marginal q. This avoids the earlier criticism of weak proxies for "desired capital stock" and identification problems in calculations of average q.

The decomposition of the opportunity set yields a two-equation simultaneous system which determines investment behavior. Simultaneous-equation estimation techniques should be brought to bear in testing the hypotheses of the model presented here. That work should shed light on suspected single-equation specification bias in earlier econometric analyses of investment.

REFERENCES

1 Abel, Andrew B., <u>Investment Theory: An Integration of Two Approaches</u>, unpublished Ph.D. thesis, MIT, 1978.

2 Arrow, K.J., "Optimal Capital Policy with Irreversible Investment," in J.N. Wolfe, ed., <u>Value, Capital and Growth</u>, University Press, Edinburgh, 1968.

3 Black, F., "Equilibrium in the Creation of Investment Goods Under Uncertainty," in M.C. Jensen, ed., <u>Studies in the Theory of Capital Markets</u>, Praeger Publ., New York, 1972.

4 Brainard, W.C. and J. Tobin, "Pitfalls in Financial Model Building," <u>American Economic Review: Papers and Proceedings</u>, Vol. 58, May 1968, pp. 59-122.

5 Chenery, H.B., "Overcapacity and the Acceleration Principle," <u>Econometrica</u>, Vol. 20, Jan. 1952, pp. 1-28.

6 Ciccolo, J.H., Jr., <u>Four Essays on Monetary Policy</u>, unpublished Ph.D. thesis, Yale University, 1975.

7 Clark, J.M., "Business Acceleration and the Law of Demand: A Technical Factor in Economic Cycles," <u>Journal of Political Economy</u>, Vol. 25, No. 1, March 1917, pp. 217-35.

8 Craine, R., "Investment, Adjustment Costs, and Uncertainty," <u>International Economic Review</u>, Vol. 16, No. 3, Oct. 1975, pp. 648-61.

9 Elton, E.J. and M.J. Gruber, "Valuation and Asset Selection Under Alternative Investment Opportunities," <u>Journal of Finance</u>, Vol. 31, No. 2, May 1976, pp. 525-39.

10 Fama, E.F., "Perfect Competition and Optimal Production Decisions Under Uncertainty," <u>Bell Journal of Economics</u>, Vol. 3, No. 2, Autumn 1972, pp. 509-30.

11 Fisher, I., <u>The Theory of Interest</u>, Macmillan and Co., New York, 1930.

12 Gould, J.P., "Adjustment Costs in the Theory of Investment of the Firm," <u>Review of Economic Studies</u>, Vol. 35, 1968, pp. 47-55.

13 Grunfeld, Y., "The Determinants of Corporate Investment," in A.C. Harberger, ed., <u>The Demand for Durable Goods</u>, University of Chicago Press, Chicago, 1960.

14 Hall, R.E., "Investment, Interest Rates, and the Effects of Stabilization Policies," <u>Brookings Papers on Economic Activity, 1: 1977</u>, the Brookings Institution, Washington, D.C., pp. 61-103.

15 Holland, D.M., and S.C. Myers, "Trends in Corporate Profitability and Capital Costs," in R. Lindsay, ed., <u>The Nation's Capital Needs: Three Studies</u>, Committee for Economic Development, New York, 1979.

16 Jensen, M. and Long, J., "Corporate Investment Under Uncertainty and Pareto Optimality in the Capital Markets," Bell Journal of Economics, Vol. 3, No. 1, Spring 1972, pp. 151-74.

17 Jorgenson, D.W., "Econometric Studies of Investment Behavior: A Survey," Journal of Economic Literature, Vol. 9, 1971, pp. 1111-47.

18 _____, "Investment Behavior and the Production Function," Bell Journal of Economics, Vol. 3, No. 1, Spring 1972, pp. 220-51.

19 _____, and C.D. Siebert, "A Comparison of Alternative Theories of Corporate Investment Behavior," American Economic Review, Vol. 58, No. 4, Sept. 1968, pp. 681-712.

20 _____, and J.A. Stephenson, "Issues in the Development of the Neoclassical Theory of Investment Behavior," Review of Economics and Statistics, Vol. 51, No. 3, Aug. 1969, pp. 346-53.

21 Keynes, J.M., The General Theory of Employment, Interest, and Money, Harcourt, Brace a World, Inc., New York, 1964.

22 Koyck, L.M., Distributed Lags and Investment Analysis, Amsterdam, 1954.

23 Kuh, E., Capital Stock Growth: A Micro-Econometric Approach, North-Holland Co., Amsterdam, 1971.

24 Lintner, J.V., "Corporation Finance: Risk and Investment," in R. Ferber, et al., Determinants of Investment Behavior, NBER-Columbia Univ. Press, 1967, pp. 215-54.

25 _____, "Optimum or Maximum Corporate Growth Under Uncertainty," in R. Marris and A. Wood, ed., The Corporate Economy, Harvard Univ. Press, Cambridge, 1971, pp. 172-241.

26 Lorie, H. and K.R. Smith, "Uncertainty, Investment and Adjustment Costs, European Economic Review, Vol. 2, No. 1, Fall 1970, pp. 31-44.

27 Merton, R.C., "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," Journal of Economic Theory, Dec. 1971.

28 _____, and Subrahmanyam, M., "The Optimality of a Competitive Stock Market," Bell Journal of Economics, Vol. 5, No. 1, Spring 1974, pp. 145-70.

29 Myers, S.C., "Procedures for Capital Budgeting Under Uncertainty," Industrial Management Review, Vol. 9, No. 3, Spring 1968, pp. 1-19.

30 _____, "Interactions of Corporate Financing and Investment Decisions--Implications for Capital Budgeting," in Modern Developments in Financial Management, ed. by S.C. Myers, Praeger, New York, 1976, pp. 347-71.

31 Oniki, H., "Comparative Dynamics (Sensitivity Analysis) in Optimal Control Theory," Journal of Economic Theory, Vol. 6 (1973), pp. 265-83.

32 Paddock, J.L., "Firm Investment Under Uncertainty with Costs of Adjustment," unpublished Ph.D. thesis, M.I.T., 1978.

33 Rothschild, M., "Essays in Economic Theory," unpublished Ph.D. thesis, M.I.T. 1969.

34 Thomadakis, S.B., "A Model of Market Power, Valuation and the Firm's Returns," Bell Journal of Economics, Vol. 7, Spring, 1976, pp. 150-161.

35 Tobin, J., "A General Equilibrium Approach to Monetary Theory," Journal of Money, Credit, and Banking, Vol. 1, Feb. 1969, pp. 15-29.

36 Von Furstenberg, G.M., "Corporate Investment: Does Market Valuation Matter in the Aggregate?" Brookings Papers on Economic Activity, 2: 1977, pp. 347-97.

FOOTNOTES

*   Author information.

1. The earlier fixed-accelerator model of Clark [7] held that investment is simply a constant proportion of changes in output. Lintner [24] gives a lucid summary of the development of the fixed-accelerator to the flexible-accelerator model. For a derivation and discussion of the distributed lag (lead) form of this model, see Jorgenson and Siebert [19] or Jorgenson and Stephenson [20].

2. For a discussion of the important distinctions between the different concepts of desired capital stock see Jorgenson [17]. Jorgenson and Siebert [19] present an excellent comparison of these methods. Gould [12] shows rigorously the unrealistic special case for which the static conditions hold.

3. Keynes [21], p. 151, where he states "the daily revaluations of the Stock Exchange, though they are primarily made to facilitate transfers of old investments between one individual and another, inevitably exert a decisive influence on the rate of current investment. For there is no sense in building up a new enterprise at a cost greater than that at which a similar existing enterprise can be purchased; whilst there is an inducement to spend on a new project what may seem an extravagant sum, if it can be floated on the Stock Exchange at an immediate profit."

4. Grunfeld [13], p. 211.

5. Abel [1] presents a comparative analysis of the standard, neoclassical model and Tobin's q model, and shows the specific role of policy variables, such as the investment tax credit.

6. Lintner [24], p. 218.

7. For a discussion of adjustment costs and the relevant literature see Gould [12] or Rothschild [33].

8. If $I \equiv$ gross investment and $s \equiv$ depreciation rate, then $\dot{K} = I - sK$.
    However, for simplicity of exposition we set $s = 0$ and consider only $\dot{K}(=I)$;
    no generality is lost through this assumption as shown by Arrow [2].

9. Our assumption that the net revenue function has positive but strictly
    diminishing returns to capital at any moment of time follows the Arrow [2]
    model.

10. For further simplicity, we assume that $\rho_t \overset{t}{=} \rho$ and $P_t \overset{t}{=} P$; that is, these
    parameters are constant over time. No generality is lost by these
    assumptions for purposes of our analysis.

11. Our second boundary condition comes from the transversality condition. A
    sufficient condition for V to be bounded from above is

    $[\int_0^\infty R(.)e^{-\rho t}dt] < \infty$. Obviously V is bounded from below by $V \geq 0$ due to

    limited liability. For a proof along these lines, see Rothschild [33, p. 7].

12. The definition of replacement cost is ambiguous in the literature. Tobin
    originally seemed to treat it as today's construction cost of existing
    assets. For now we simply consider it as that equilibrium price which would
    be paid in a secondary market for a unit of the firm's assets.

13. Of course an efficient market anticipates that the firm will take all
    investment opportunities which have NPV > 0. Thus this value increment is
    reflected in the firm's market value when recognized, and the later act of
    actually taking the investment will have NPV = 0, as discussed above.

14. Without adjustment costs, all the terms involving the function G(k) go to
    zero in equation (6). The firm invests in projects, accepting lower R_K
    until (6) is satisfied at q' = 1.

15. Recall that q' is a decreasing function of the level of K. Thus as
    additional units are added via investment, q' will fall.

16. Recall our simplifying assumption that $\dot{P} = 0$.

17. The discrete changes illustrated for expositional purposes in Figure 3 can only correspond to, rather than directly represent, the rates of change in our continuous time mathematical analysis. Nor does the Figure depict the path of dynamic adjustment, e.g., the distributed lead on investment where there exist costs of adjustment.

18. There is no loss of generality for strictly convex functions for purposes of our analysis by this assumption.

19. For a rigorous discussion of the assumptions embedded in this procedure see Oniki [31]. Our model satisfies those requirements.

20. Abel [1] was the first to incorporate adjustment cost properties into phase diagrams in his analysis of tax incentives.

BASEMENT